Title: Mix of Rational Expressions to Simplify and Rational Equations to Solve
Class: Math 100 or Math 107 or Math 111
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Instructions to Tutor: Read instructions and follow all steps for each problem exactly as given.
Keywords/Tags: adding rational expressions, subtracting rational expressions, rational expressions, solving rational equations, rational equations

Mix of Rational Expressions to Simplify and Rational Equations to Solve

Purpose: This is intended to refresh your skills in simplifying rational expressions and solving rational equations, and how to tell the difference.

Activity: Work through the following activity and examples. Do all of the practice problems before consulting with a tutor.

Sometimes students get confused when confronted with lots of impossible-looking rational expressions whether they are supposed to "simplify" or "solve." To make things worse, we seem to use different rules to "simplify" versus "solve," and the expressions all look the same, and we seem to use the LCD (least common denominator) in both, but we use it differently. It is all so confusing! How can a student tell the difference? The answer is surprisingly easy:

Is there an " = " in the expression?

- ## NO

  This means the expression is just an expression to be simplified. We can not solve it if there is no equation to solve!

  We may only use the rules of simplification, which basically means that we can only multiply by 1, since multiplying by one does not change the value or meaning of the expression. The hard part is to use the version of 1 which helps you simplify or work with the expression. In reducing, 1 is a factor divided by the same factor. In getting a common denominator (prior to adding or subtracting), 1 is the "missing" factor from the denominator, then multiplied by the same factor (this is "un-reducing").

- ## YES

  This means the expression is actually an equation that can be solved.

  In solving equations we are given a lot more freedom in the operations we can perform. In the case of rational equations, in general the most efficient thing to do is to multiply both sides of the equation by the LCD. Then we will be able to reduce out each denominator, and the resulting equation will no longer have fractions!

Example 1a $\frac{3m}{2}+\frac{8-4m}{7}-3$

There is no =, so this is a **simplify** problem.

The LCD is 14.

$\frac{3m}{2}\bullet\frac{7}{7}+\frac{8-4m}{7}\bullet\frac{2}{2}-3\bullet\frac{14}{14}$ Multiply each term by missing denominator factor of 14.

$\frac{(3m)(7)+(8-4m)(2)-(3)(14)}{14}$ Denominators are the same, so rewrite as one fraction.

$\frac{21m+16-8m-42}{14}$ Simplifying the numerator. **This is still a fraction.**

$\frac{13m-26}{14}$

$\frac{13(m-2)}{14}$ Factor the numerator to see if the fraction is reducible.

Example 1b $\frac{3m}{2}+\frac{8-4m}{7}=3$

There is an =, so this is a **solve** the equation problem.

The LCD is 14. No restrictions for denominators.

$\frac{14}{1}\bullet\frac{3m}{2}+\frac{14}{1}\bullet\frac{8-4m}{7}=\frac{14}{1}\bullet\frac{3}{1}$ Multiply each term in the = by the entire LCD, not just the missing factor in the denominator.

$\frac{7\,\cancel{14}}{1}\bullet\frac{3m}{\cancel{2}}+\frac{2\,\cancel{14}}{1}\bullet\frac{8-4m}{\cancel{7}}=\frac{14}{1}\bullet\frac{3}{1}$ Reduce out the denominators.

$(7)(3m)+(2)(8-4m)=(14)(3)$ Now there are no more denominators. Solve the =.

$21m+16-8m=42$ Notice how similar this line is to step 4 in Example 1a. But there are no more fractions in this equation to solve.

$13m=58$

$m=\frac{58}{13}$

Practice 1a $\frac{2}{9}-\frac{2x-3}{6}+\frac{x-1}{2}$

Practice 1b $\frac{2}{9}=\frac{2x-3}{6}+\frac{x-1}{2}$

Did you get $\frac{3m+4}{18}$?

Did you get $m=\frac{22}{15}$?

Example 2a $\frac{3}{2m-6}-\frac{2m-3}{2m^2-5m-3}+\frac{1}{4m+2}$ **LCD** is $2(2m+1)(m-3)$.

$$\frac{3}{2(m-3)}\bullet\frac{(2m+1)}{(2m+1)}-\frac{2m-3}{(2m+1)(m-3)}\bullet\frac{2}{2}+\frac{1}{2(2m+1)}\bullet\frac{(m-3)}{(m-3)}$$

$$\frac{3(2m+1)-2(2m-3)+1(m-3)}{2(2m+1)(m-3)}$$

$$\frac{6m+3-4m+6+m-3}{2(2m+1)(m-3)}$$

$$\frac{3m+6}{2(2m+1)(m-3)}$$

$$\frac{3(m+2)}{2(2m+1)(m-3)}$$

Example 2b $\frac{3}{2m-6}-\frac{2m-3}{2m^2-5m-3}+\frac{1}{4m+2}=0$ **LCD** is $2(2m+1)(m-3)$.

Restrictions are $m\neq 3,\ -\frac{1}{2}$.

$$\frac{2(2m+1)(m-3)}{1}\bullet\frac{3}{2(m-3)}-\frac{2(2m+1)(m-3)}{1}\bullet\frac{2m-3}{(2m+1)(m-3)}+\frac{2(2m+1)(m-3)}{1}\bullet\frac{1}{2(2m+1)}=\frac{2(2m+1)(m-3)}{1}\bullet 0$$

$$3(2m+1)-2(2m-3)+1(m-3)=0$$

$$6m+3-4m+6+m-3=0$$

$$3m+6=0$$

$$m=-2$$

For the following problems designated with the *****, recall the following information:

- When terms are being added, we can rewrite their order using the commutative property of addition: $3+x=x+3$
- But, subtraction is not commutative: $3-x\neq x-3$
- However, we can factor out a -1: $3-x=-1(-3+x)=-1(x-3)$

  So, when you need to rewrite the order of two terms being subtracted, factor out a -1.
- Use the following properties to rewrite where you put your negative signs (by convention, we try not to leave any in denominators).

$$\frac{A}{-B}=\frac{-A}{B}=-\frac{A}{B} \quad \text{and} \quad -\frac{A}{-B}=-\frac{-A}{B}=\frac{-A}{-B}=\frac{A}{B}$$

Problems

1a) $\frac{11}{12}-\frac{3}{2x}+\frac{4}{x}$

1b) $\frac{11}{12}-\frac{3}{2x}=\frac{4}{x}$

2a) $\frac{a}{2}+\frac{a-6}{3a-9}-\frac{1}{3}$

2b) $\frac{a}{2}=\frac{a-6}{3a-9}-\frac{1}{3}$

3a) $\frac{1}{6x}-\frac{2}{x-6}$

3b) $\frac{1}{6x}=\frac{2}{x-6}$

4a)***** $\frac{2x-5}{2-x}+\frac{x}{2x-4}$

4b)***** $\frac{2x-5}{2-x}=\frac{x}{2x-4}$

5a) $\frac{1}{a+7}+\frac{a}{a^2-49}$

5b) $\frac{1}{a+7}=\frac{a}{a^2-49}$

6a)***** $\frac{3}{x^2-25}-\frac{1}{5+x}-\frac{x+1}{5-x}$

6b)***** $\frac{3}{x^2-25}=\frac{1}{5+x}-\frac{x+1}{5-x}$

7a) $4k-\frac{k+1}{k-1}+3$

7b) $4k-\frac{k+1}{k-1}=3$

8a) $\frac{2}{3m+12}-\frac{1}{9m-3}-\frac{m-2}{3m^2+11m-4}$

8b) $\frac{2}{3m+12}-\frac{1}{9m-3}-\frac{m-2}{3m^2+11m-4}=0$

Review: Meet with a tutor to verify your work on this worksheet and discuss some of the areas that were more challenging for you. If necessary, choose more problems from the homework to practice and discuss with the tutor.

For Tutor Use: Please check the appropriate statement:

____________Student has completed worksheet but may need further assistance. Recommend a follow-up with the instructor.

____________Student has mastered topic.